	P.R.Government College (Autonomous) KAKINADA	**Program&Semester** III B.Sc. Mathematics Major & Statistics, Chemistry Minors (V Sem)
CourseCode **MAT- 502 T**	**TITLEOFTHECOURSE** **Vector Calculus & Problem Solving Sessions**	

		L	T	P	C
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on Integration	3	-	-	3

Course Objectives:

This course will cover the particular mathematical functions that have more or less established names and notations due to their importance in mathematical analysis, functional analysis, geometry, physics, or other applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Learn multiple integrals as a natural extension of definite integral to a function of two variables in the case of double integral/three variables in the case of triple integral.
CO2	Learnapplicationsintermsoffindingsurfaceareabydoubleintegralandvolumebytripleintegral.
CO3	Determine the gradient, divergence and curl of a vector and vector identities.
CO4	Evaluate line, surface and volume integrals.
CO 5	understand relation between surface and volume integrals (Gauss divergence theorem), relation between line integral and volume integral(Green's theorem), relation between line and surface integral(Stokes theorem)

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

Syllabus:

Unit – 1: MultipleIntegrals-I.

Introduction -Double integrals -Evaluation of double integrals –Properties of double integrals -Region of integration -double integration in Polar Co-ordinates –Change of variables in double integrals -change of order of integration.

Unit– 2: Multipleintegrals-II

Triple integral -region of integration -change of variables -Plane areas by double integrals -

Surface area by double integral -Volume as a double integral, volume as a triple integral.

Unit – 3: Vector differentiation

Vector differentiation –ordinary – derivatives of vectors – Differentiability –Gradient –Divergence -

Curl operators – Formulae involving the separators.

Unit – 4: Vector integration

1. Line Integrals with examples - Surface Integral with examples – Volume integral with examples.

Unit – 5: Vector integration applications

1. Gauss theorem and applications of Gauss theorem - Green's theorem in plane and Applications of Green's theorem - Stokes's theorem and applications of Stokes theorem.

II. Reference Books:

Text Book

A text Book of Higher Engineering Mathematics by B.S.Grawal, Khanna Publishers, 43 rd Edition

ReferenceBooks

1. Vector Calculus by P.C.Matthews, Springer Verlag publications.
2. Vector Analysis by Murray Spiegel, Schaum Publishing Company, NewYork

III. Co-Curricular Activities:

Seminar/ Quiz/ Assignments/ Applications of Vector calculus to Real life Problems /Problem Solving Sessions.

<u>BLUE PRINT FOR QUESTION PAPER PATTERN,</u>

Paper –Major XIII & Minor VI : **Vector Calculus & Problem solving Sessions**

UNIT	TOPIC	S.A.Q	E.Q	Marks Allotted
I	Multiple Integrals-I	02	01	20
II	Multiple Integrals-II	02	01	20
III	Vector differentiation	01	01	15
IV	Vector integration	01	01	15
V	Vector integration applications	01	02	25
Total		07	06	95

S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (10 marks)

Short answer questions : 4 x 5 M = 20
Essay questions : 3 x 10 M = 30

Total Marks : = 50

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Pithapur Rajah's Government College (Autonomous), Kakinada

III Year B.Sc., Degree Examinations - V Semester

Mathematics Course: Major XIII & Minor VI : Vector Calculus & Problem solving Sessions

(Model Paper w.e.f. 2025-26)

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Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer Any Three Questions, Selecting At Least One Question From Each Part

Part – A **3 X 10 = 30**

1.Essay question from Unit - I.

2. Essay question from Unit – II

3. Essay question from Unit - III.

Part – B

4.Esay question from Unit - IV.

5.Essay question from Unit - V.

6. Essay question from Unit - V .

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

1. Short answer question from Unit - I.
2. Short answer question from Unit - I .
3. Short answer question from Unit – II.
4. Short answer question from Unit – II.
5. Short answer question from Unit - III.
6. Short answer question from unit – IV.
7. Short answer question from Unit – V.

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PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
Question Bank for
PAPER–Major XIII & Minor VI : Vector Calculus & Problem solving Sessions
Short Answer Questions

Unit-I

1. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\ dx\, dy$
2. Evaluate $\iint xy(x + y)\, dx\, dy$ over the area between y = x^2 and y = x.
3. Evaluate $\iint (x^2 + y^2)\, dx\, dy$ over the area bounded by x + y ≤ 1in the first quadrant.
4. Evaluate $\iint r\sqrt{a^2 - r^2}\, d\theta\, dr$ over the upper half of the circle r = a cosθ.
5. By changing into polar coordinates evaluate the integral $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\, dx\, dy$
6. Change the order of integration in the double integral $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\, dx\, dy$ and hence find the value.

Unit-II

7. Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\, dx\, dy\, dz$.
8. Evaluate $\iiint_V (x^2 + y^2 + z^2)\, dx\, dy\, dz$ where V is the volume of the cube bounded by the coordinate planes and the planes x = y = z = a.
9. Find the smaller of the areas bounded by y = 2 – x and $x^2 + y^2 = 4$ using double integral.
10. Find the area of a loop of the curve t = a sin3θ.
11. Find the area of the surface of the sphere of radius r.
12. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

Unit-III

13. If $\boldsymbol{r} = e^{-t} i = \log(t^2 + 1)\, j - \tan t\ \ k$ then find

$$i)\ \frac{d\boldsymbol{r}}{dt}, \frac{d^2\boldsymbol{r}}{dt^2}, \left|\frac{d\boldsymbol{r}}{dt}\right|, \left|\frac{d^2\boldsymbol{r}}{dt^2}\right|\ at\ t = 0 \quad ii)\ \left(\frac{d\boldsymbol{r}}{dt} \times \frac{d^2\boldsymbol{r}}{dt^2}\right) at\ t = 0$$

14. If $\emptyset = 2xz^4 - x^2y$ then find $\left|\frac{\partial \emptyset}{\partial x} i + \frac{\partial \emptyset}{\partial y} j + \frac{\partial \emptyset}{\partial z} k\right|$ at (2,-2,-1).
15. Find the directional derivative of f = $x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line $\overrightarrow{PQ}$ where Q = (5,0,4).
16. Find div F and Curl F where $F = xy^2 i + 2x^2yz j - 3yz^2 k$ at (1,-1,1).
17. If $F = grad(x^3 + y^3 + z^3 - 3xyz)$ then find div F, curl F.

Unit-IV

18. If $A = t\, i - t^2 j + (t - 1)k, B = 2t^2 i + 6t\ k$ find $i)\ \int_0^2 (A.B)\, dt \quad ii)\ \int_0^2 (A \times B)\, dt$

19. If $F = x^2y^2\ i + yj$, evaluated $\int_c F.dr$ where c is the curve $y^2 = 4x$ in the xy – plane from (0, 0) to (4, 4).
20. Evaluate $\int_c F.dr$ where $F = 3x^2\ i + (2xz - y)\ j + z\ k$ along the straight line C from (0,0,0,) to (2,1,3).
21. Evaluate $\int_S F.N\ ds$ where $F = z\ i + x\ j - 3y^2\ z\ k$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5.
22. If $\emptyset = 45x^2y$, evaluate $\iiint_V \emptyset\ \ dv$ where V is the closed region bounded by the plane 4x + 2y + z = 8, x = 0, y = 0, z = 0.

Unit-V

23. If F = ax i + by j + cz k and a, b, c are constants. Show that $\int F.N\ ds = \frac{4\pi}{3}(a + b + c)$ where S is the surface of the unit sphere.
24. Compute $\int_S (ax^2 + by^2 + cz^2)\ ds$ over the sphere $x^2 + y^2 + z^2 = 1$.
25. State and prove Green's theorem in a plane.
26. Evaluate $\int_c (\cos x\ \ \sin y - xy)dx + \sin x\ \cos y\ dy$, by Green's theorem where C is the circle $x^2 + y^2 = 1$.
27. Evaluate $\int_S \nabla \times F.N\ ds$ using Stoke's theorem, where $F = (2x - y)i - yz^2j - y^2z\ k$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

Essay Questions

Unit-I

1. Evaluate $\iint (x + y)^2\ dx\ dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
2. Change into polar coordinates and evaluated $\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\ dx\ dy$
3. By changing into polar coordinates evaluate $\iint \frac{x^2y^2}{x^2+y^2} dx\ dy$ over the angular region between the circles $x^2 + y^2 = a^2, x^2 + y^2 = b^2 (b > a)$.
4. By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y)\ dx\ dy$.

Unit-II

5. Evaluate $\iiint_V (x^2 + y^2 + z^2)\ dx\ dy\ dz$ taken over the volume enclosed by the sphere $x^2 + y^2 + z^2 = 1$.
6. Find the area inside the circle r = a sinθ but lying outside the cardiod r = a(1 - cosθ).
7. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by using double integral.

8. Find the volume of the tetrahedron bounded by coordinates planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

Unit-III

9. If $\boldsymbol{r} = a\cos t\ i + a\sin t\, j + at\tan\theta\, k$ then find i) $\left(\frac{d\boldsymbol{r}}{dt}\times\frac{d^2\boldsymbol{r}}{dt^2}\right) at\ t = 0$ ii) $\left|\frac{d\boldsymbol{r}}{dt}\times\frac{d^2\boldsymbol{r}}{dt^2}\right|$ and iii) $\left[\frac{d\boldsymbol{r}}{dt}\ \frac{d^2\boldsymbol{r}}{dt^2}\ \frac{d^3\boldsymbol{r}}{dt^3}\right]$
10. If A, B are two differentiable vector point functions then $grad(A.B) = (B.\nabla)A + (A.\nabla)B + B\times curl\ A + A\times curl\ B$.
11. If A, B are two differentiable vector point functions then $div(A\times B) = B.curl\ A - A.curl\ B$.
12. If A, B are two differentiable vector point functions then $Curl(A\times B) = A\ div\ B - B\ div\ A + (B.\nabla)A - (A.\nabla)B$
13. If F is a differentiable vector point function, then $\nabla\times(\nabla\times F) = \nabla(\nabla.F) - \nabla^2 F$.

Unit-IV

14. Evaluate $\int_S F.N\,ds$ where F = 18 z i – 12 j + 3y k and S is the part of the plane 2x + 3y + 6z = 12 located in the first octant.
15. If F = (x + y^2) i – 2x j + 2yz k, evaluate $\int_S F.N\,ds$ where S is the surface of plane 2x + y + 2z = 6 in the first octant.
16. If F = 2xz i – x j + y^2 k, evaluate $\iiint_V F\,dv$ where V is the region bounded by the surface x = 0, y = 0, y = 6, z = x^2, z = 4.
17. If F = (2x^2 – 3z) i – 2xy j – 4x k, then evaluate $\iiint_V \nabla.F\,dv$ where v is closed region bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y +z = 4. Also evaluate $\iiint_V \nabla\times F\,dv$.

Unit-V

18. State and prove Gauss's divergence theorem.
19. Verify Gauss's divergence theorem to evaluate $\int_S\{(x^3 - yz)i - 2x^2yj + zk\}.N\,ds$ over the surface of a cube bounded by the coordinate planes x = y = z = a.
20. By transforming into triple integral, evaluate $\iint_S(x^3\,dy\,dz + x^2y\,dz\,dx + x^2z\,dx\,dy)$ where S is the closed surface consisting of the cylinder $x^2+y^2=a^2$ and the circular disc z = 0, and z = b.
21. Verify Green's theorem in the plane for $\int_c(3x^2-8y^2)dx + (4y-6xy)dy$ where C is the region bounded by $y=\sqrt{x}\ \ and\ y = x^2$.
22. State prove Stoke's theorem.
23. Verify Stoke's theorem for F = - y^3 i + x^3 j where S is the circle disc $x^2+y^2\le 1, z = 0$.
24. Verify Stoke's theorem for F = (2x – y) i – yz^2 j – y^2z k, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is boundary.

<table>
<tr><td rowspan="1"></td><td colspan="1">P.R.Government College
(Autonomous)
KAKINADA</td><td colspan="4" rowspan="2">Program & Semester
III B.Sc. Mathematics Major & Statistics, Chemistry Minors
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<tr><td>CourseCode
MAT-502 P</td><td>TITLEOFTHECOURSE
Vector Calculus & Problem solving Sessions</td></tr>
<tr><td>Teaching</td><td>HoursAllocated:30(Practicals)</td><td>L</td><td>T</td><td>P</td><td>C</td></tr>
<tr><td>Pre-requisites:</td><td>Basic Mathematics Knowledge on Integration</td><td>-</td><td>-</td><td>2</td><td>1</td></tr>
</table>

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Semester – V End Practical Examinations

Scheme of Valuation for Practical's

Time : 2 Hours **Max.Marks : 50**

- **Record - 10 Marks**
- **Viva voce - 10 Marks**
- **Test - 30 Marks**
- **Answer any 5questions. At least 2 questions from each section. Each question carries 6 marks.**

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COURSE-Major XIII & Minor VI : Vector Calculus & Problem solving Sessions

Unit	TOPIC	E.Q	Marks allotted to the Unit
I	Multiple Integrals-I	2	12
II	Multiple Integrals-II	2	12
III	Vector differentiation	1	06
IV	Vector integration	2	12
V	Vector integration applications	1	06
	Total	08	48

PITHAPUR RAJAH'S GOVT. COLLEGE (AUTONOMOUS), KAKINADA

III year B.Sc., Degree Examinations - V Semester

Mathematics Course-Major XIII & Minor VI : Vector Calculus & Problem solving Sessions

(w.e.f. 2023-24 Admitted Batch)

Practical Model Paper (w.e.f. 2025-2026)

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Time: 2Hrs **Max. Marks: 50M**

Answer any 5questions. At least 2 questions from each section. **5 x 6 = 30 Marks**

SECTION - A

1. Unit - I.
2. Unit – I.
3. Unit - II.
4. Unit – II.

SECTION – B

5. Unit - III.
6. Unit - IV.
7. Unit – IV.
8. Unit - V.

- **Record - 10 Marks**
- **Viva voce - 10 Marks**